RESEARCH ARTICLE OPEN ACCESS

Thermal Instability of Chemically Reacting Maxwell Fluid in a Horizontal Porous Layer with Constant Heat Flux Lower Boundary

Syeda Khudeja Akbar*, Achala L. Nargund**, and S. Maruthamanikandan***
*(Department of Mathematics, Christ University, Hosur Road, Bengaluru – 560 029, India)
**(Postgraduate Department of Mathematics & Research Centre in Applied Mathematics, MES College, Malleshwaram, Bengaluru – 560 003, India)
*** (Department of Mathematics, Presidency University, Itgalpur, Bengaluru – 560 089, India)

ABSTRACT

The effect of chemical reaction on the linear stability of a viscoelastic fluid saturated horizontal densely-packed porous layer is investigated. The viscoelastic properties are given by Maxwell constitutive relations. The porous layer is cooled from the upper boundary while an adiabatic thermal boundary condition is imposed at the lower boundary. Linear stability analysis suggests that there is a competition between the processes of viscous relaxation and thermal diffusion that causes the first convective instability to be oscillatory rather than stationary. The effect of Deborah number, Darcy-Prandtl number, normalized porosity, and the Frank-Kamenetskii number on the stability of the system is investigated. Using a weighted residual method we calculate numerically the convective thresholds for both stationary and oscillatory instability. The effects of viscoelasticity and chemical reaction on the instability are emphasized. Some existing results are reproduced as the particular cases of the present study.

***Keywords* -** Chemical reaction, Maxwell fluid, Oscillatory instability, Porous medium, Viscoelasticity

I. Introduction

Buoyancy-driven phenomena in porous media are actively under investigation as they have a wide variety of engineering applications such as geothermal reservoirs, agricultural product storage systems, packed-bed catalytic reactors, the pollutant transport in underground and the heat removal of nuclear power plants. Convection of non-Newtonian fluids in a porous medium is of considerable importance in several applied fields such as oil recovery, food processing, soil decontamination, storage of chemical or agricultural products, the spread of contaminants in the environment and in various processes in the chemical and materials industry. The onset of thermal convection in a viscoelastic fluid was studied by many authors [1-4]. Extensive reviews on this subject can be found in the books by Nield and Bejan [5] and by Kaviany [6].

With the growing importance of composite materials made up of polymeric substances in modern technology, particularly, in material processing, nuclear engineering, geophysics and bio-engineering, the investigations of such fluids is desirable. These fluids have high molecular weight and are viscoelastic in nature. Flow instability and turbulence are far less widespread in viscoelastic fluids than in Newtonian fluids because of high viscosity of the polymeric fluids. Viscoelastic fluids are expected to show markedly different behaviours of evolving convective instabilities.

In the linear stability analysis, if the imaginary part of the largest eigenvalue at the neutrally stable state is zero, the new mode or flow pattern grows monotonically without oscillation and we say that the exchange of stability is valid. On the other hand, if the imaginary part of the largest eigenvalue at the neutrally stable state is non-zero, the new mode grows with oscillation and this instability is called overstability. As the elasticity of viscoelastic fluids allows the periodic instability to be sustained in addition to the stationary modes, viscoelastic fluids will exhibit an oscillatory convection at the threshold of stationary mode.

Since the Rayleigh-Bénard convection involves complicated flow fields that are closer to polymeric processing situations than the usual viscometric flows, investigating this problem through a viscoelastic fluid model is essential. Copious literature is available on thermal convection in a viscoelastic fluid layer heated from below and is well documented [7-8]. Nevertheless, its counterpart in a porous layer has received little attention [9-12]. Tan and Masuoka [13] performed a stability analysis of a Maxwell fluid in a porous medium heated from below based on Darcy-Brinkman-Maxwell model. Wang and Tan [14] studied the linear stability of a Maxwell fluid in the Bénard problem for a double diffusive mixture in a porous medium based on the Darcy-Maxwell model. The effect of thermal/gravity modulation on the onset of convection in a Maxwell fluid saturated porous layer was studied by Malashetty and Begum [15]. More recently, Ersoy [16] studied the steady flow of a Maxwell fluid in a porous orthogonal rheometer with the application of a magnetic field. It was observed that the effects of the Deborah number, the suction/injection velocity parameter, and the Reynolds number in the absence of a magnetic field are similar to those in the presence of a magnetic field.

When an exothermic reaction takes place in a fluid-saturated porous medium, the heat generated by the reaction changes the fluid density and free convection may occur. The induced natural convection, in turn, affects the rate of heat release by the reaction. The complex interaction between these two processes is the major cause of many of the instabilities observed in chemically reacting systems.

Kordylewski and Krajewski [17] were the first to perform a stability analysis based on Darcy's law with the Boussinesq approximation and a zero-order exothermic reaction. Farr *et al*. [18] carried out stability analysis on free convection in a confined porous medium with zero-order exothermic reactions. Malashetty *et al*. [19] performed a linear stability analysis to study the onset of convective instability in a horizontal inert porous layer saturated with a fluid undergoing a zero-order exothermic chemical reaction. McKay [20] investigated the onset of buoyancy-driven convection in superposed reacting fluid and porous layers. Khudeja *et al.* [21] investigated the problem of onset of convective instability in a horizontal inert porous layer saturated with a Maxwell viscoelastic fluid subject to zero-order chemical reaction by linear stability analysis. The possibility of oscillatory instability, which is inherent in viscoelastic fluid convection, is discussed. It is found that, with chemical reactions, the fluid in the porous medium is more prone to instability as compared to the case in which chemical reactions are absent. Recently, Rashidi *et al*. [22] employed optimal homotopy analysis method to investigate the steady laminar incompressible free convective flow of a nanofluid past a chemically reacting upward facing horizontal plate in a porous medium taking into account heat generation/absorption and the thermal slip boundary condition.

Furthermore, this problem has application in the study of mobility control in oil displacement mechanisms which improves the efficiency of oil recovery. The performance of a reservoir depends, to a large extent, upon the physical nature of crude oil present in the reservoir. The heavy crude is non-Newtonian and also some oil sands contain waxy crudes at shallow depths of the reservoirs which are considered to be viscoelastic. Hence a viscoelastic model of a fluid serves to be more realistic than any other Newtonian or non-Newtonian model.

The purpose of the present work is to analyze the influence of viscoelasticity on Rayleigh-Bénard convective thresholds in a Maxwell fluid when the fluid is undergoing a zero order exothermic chemical reaction and lower boundary is adiabatic in nature. In the neighborhood of the critical conditions, the effect of relaxation parameter on the variation of chemical reaction parameter with respect to media Darcy-Rayleigh number is investigated. This problem will be promising in designing and operating many processes of non-Newtonian liquids in porous materials involving natural convection that cannot be explained for the stationary mode.

II. Mathematical Formulation

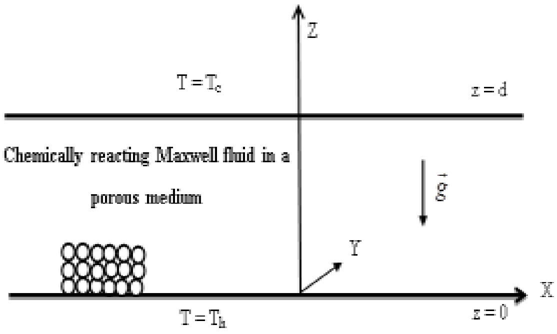

Figure 1. Physical configuration of the problem.

We consider an incompressible viscoelastic fluid saturated horizontal porous layer of finite thickness *d*. A Cartesian co-ordinate system (*x*, *y*, *z*) is chosen such that the origin is at the bottom of the layer and the *z*-axis is directed vertically upward. The boundaries of the porous layer are kept at constant but different temperatures, that is, T_h at the lower boundary $z = 0$, and $T_c\,(<T_h)$ at the upper boundary $z = d$.

The inert porous layer contains a chemically reactive Maxwell fluid which is subject to weakly exothermic chemical reactions. If the temperature in the whole domain of interest varies slightly from T_c, a zero order reaction can be assumed. Moreover, it is assumed that local thermal equilibrium exists between the solid matrix and the saturated fluid. The Boussinesq approximation, which states that the effect of compressibility is negligible everywhere in the conservation equations except in the buoyancy term, is assumed to be valid and the rheology of the viscoelastic fluid is approximated by the Maxwell constitutive model. The system of equations describing the problem under consideration is the following

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\left(1+\bar{\lambda}\frac{\partial}{\partial t}\right)\frac{\rho_0}{\varepsilon}\left(\frac{\partial \vec{q}}{\partial t}\right) = \left(1+\bar{\lambda}\frac{\partial}{\partial t}\right)\left[-\nabla p+\rho\,\vec{g}\,\right]-\frac{\mu}{k}\vec{q} \qquad (2)$$

$$\gamma\,\frac{\partial T}{\partial t}+\left(\vec{q}\cdot\nabla\right)T=\kappa\nabla^2 T+Q\,e^{-\frac{E}{RT}} \qquad (3)$$

$$\rho=\rho_0\left[1-\alpha\left(T-T_c\right)\right] \qquad (4)$$

where $\vec{q}=(u,v,w)$ is the mean filter velocity, *t* is the time, *p* is the pressure, ρ is the fluid density, $\bar{\lambda}$ is the stress relaxation time, ρ_0 is the reference density, $\vec{g}$ is the acceleration due to gravity, μ is the fluid viscosity, ε is the porosity, k is the permeability of the porous medium, κ is the effective thermal diffusivity, γ is the ratio of the specific heat of the solid due to porous medium and that of the fluid at constant pressure, α is the thermal expansion coefficient, T is the temperature, Q is the product of the heat of reaction, a pre-exponential factor and reactant concentration, *E* is the activation energy, *R* is the universal gas constant, ∇ is the vector differential operator and (x,y,z) are the spatial coordinates.

The thermal boundary conditions are given by

$$T(x,y,d)=T_c \qquad (5)$$

and

$$\frac{\partial T}{\partial z}(x,y,0)=0\,, \qquad (6)$$

where $T_h>T_c$. The fluid in the porous medium is subject to high-activation energy such that $RT_c/E<<1$. With this approximation, Eq. (3) simplifies to

$$\gamma\,\frac{\partial\theta}{\partial t}+\left(\vec{q}\cdot\nabla\right)\theta=\kappa\,\nabla^2\theta+\delta\,exp(\theta), \qquad (7)$$

where $\delta=Q\,exp\left(-E/RT_c\right)/T_r$, $\theta=\left(T-T_c\right)/T_r$ and $T_r=RT_c^2/E$. Here T_r is the prescribed reference temperature for the reacting fluid. Eqs. (5) and (6), in terms of θ, reduce to

$$\theta=0 \quad \text{at} \quad z^*=1 \qquad (8)$$

and

$$\frac{\partial\theta}{\partial z}=0 \quad \text{at} \quad z^*=0 \qquad (9)$$

At an undisturbed state, we have

$$\vec{q}=(0,0,0)\,,\ \theta=\theta_b(z)\,,\ p=p_b(z)\,,\ \rho=\rho_b(z) \qquad (10)$$

The quiescent state solutions are therefore given by the following equations

$$\frac{dp_b}{dz}-\rho_b\,g=0 \qquad (11)$$

$$\rho_b=\rho_o\left[1-\alpha\,T_r\,\theta_b\right] \qquad (12)$$

$$\kappa\,\frac{d^2\theta_b}{dz^2}+\delta\,exp\left(\theta_b\right)=0\,. \qquad (13)$$

Eq. (13) can be rewritten in the dimensionless form

$$\frac{d^2\theta_b}{dz^{*2}}=-F\,exp\left(\theta_b\right), \qquad (14)$$

where $F=\delta d^2/\kappa$. The dimensionless number F is the Frank-Kamenetskii number which is the ratio of characteristic flow time to characteristic reaction time. On integration, Eq. (14) leads to (after dropping the asterisk) the general solution

$$\theta_b=\log\left(\frac{C_1}{2F}\right)+\log\left[1-\left(\frac{1-C_2\,e^{-\sqrt{C_1}\,z}}{1+C_2\,e^{-\sqrt{C_1}\,z}}\right)^2\right] \quad for\ F\neq 0 \qquad (15)$$

where C_1 and C_2 are the integration constants to be determined. Application of the following boundary conditions

$$\frac{d\theta_b}{dz^*}=\theta_h \quad \text{at} \quad z^*=0 \quad \text{and} \quad \theta_b=0 \quad \text{at} \quad z^*=1 \qquad (16)$$

gives C_1 implicitly through the following equation

$$e^{\sqrt{C_1}}\left[1-\sqrt{1-\frac{2F}{C_1}}\right]=\left[1+\sqrt{1-\frac{2F}{C_1}}\right] \qquad (17)$$

and

$$C_2=1. \qquad (18)$$

It follows from (17) that there exists an implicit relation between C_1 and F. This relation is illustrated in Fig. 2, which shows that no solution exists for C_1 beyond the critical value of F. For F less than this critical value, there are two values of C_1 which correspond to two basic undisturbed temperature profiles. This value of F beyond which no solution exists is called the ignition value [19].

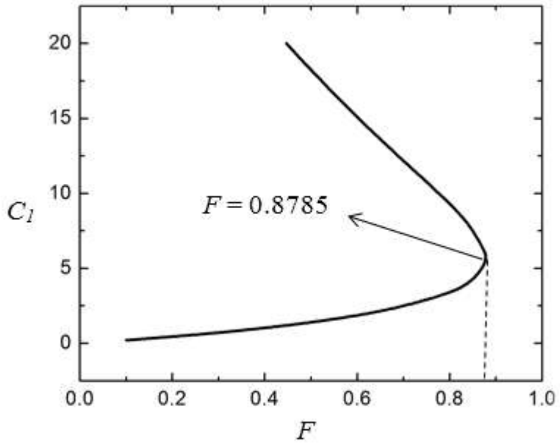

Figure 2. Plot of C_1 versus F.

For values of F higher than this critical value, the solution jumps to combustion state. The ignition point corresponds to adiabatic conditions at the lower boundary

$$\frac{\partial F}{\partial C_1}=0. \tag{19}$$

Applying (19) to (17) gives value of C_1 which on further substitution into (17) gives

$$e^{\sqrt{2F+4}}=\frac{\sqrt{2F+4}+2}{\sqrt{2F+4}-2} \tag{20}$$

A numerical solution of the above equation gives $F=0.8785$, which is the critical value of F at ignition for this fluid with a constant heat flux lower boundary. The ignition value obtained here is the same as the one obtained by Mubeen *et al.* [23] though the lower boundary was considered to be isothermal in their case.

The values of C_1 for a physically realistic case (corresponding to the lower branch of C_1) at different values of F are shown in Table 1.

Table 1: Values of C_1 for different values of F

F	C_1
0.1	0.210722
0.2	0.446317
0.3	0.71355
0.4	1.02254
0.5	1.38951
0.6	1.84335
0.7	2.44491
0.8	3.37523
0.8785	5.75696

The temperature of the adiabatic lower wall in the basic state can be obtained from (15) which on substituting into (17) gives

$$F=\left(\frac{1}{2e^{\theta_h}}\right)\left[\ln\left(\frac{1-\sqrt{1-e^{\theta_h}}}{1+\sqrt{1-e^{\theta_h}}}\right)\right]^2 \tag{21}$$

Computations for C_1 for physically realistic cases were performed for selected values of θ_h and for different values of F up to its ignition value. The results of these computations for $\theta_h=1$ are presented in Fig. 3.

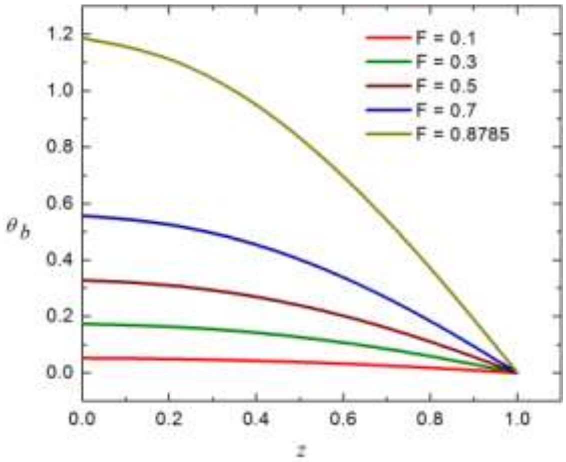

Figure 3. Basic temperature profiles for different values of F and $\theta_h=1$.

It is seen that, for small values of F, the basic temperature profile is piecewise linear, the different gradients result from the different thermal conductivities in each region. However, the basic temperature profile turns out to be more and more nonlinear, indicating a stronger chemical reaction, as the value of F is increased as the heat generated due to the chemical reaction also increases.

III. Stability Analysis

We perform a linear stability analysis by letting

$$\left.\begin{aligned} \vec{q} &= \left(u', v', w'\right) \\ p &= p_b(z) + p'(x,y,z,t) \\ \rho &= \rho_b(z) + \rho'(x,y,z,t) \\ \theta &= \theta_b(z) + \theta'(x,y,z,t) \end{aligned}\right\} \tag{22}$$

where the primes indicate infinitesimally small perturbations from the undisturbed state. On substituting (22) into (1), (2), (4) and (7), neglecting the nonlinear terms, incorporating the quiescent state solutions and eliminating the pressure term, we obtain the following equations

$$\begin{aligned} &\frac{\rho_0}{\varepsilon}\left(1+\bar{\lambda}\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\left(\nabla^2 w'\right) \\ &\quad = \rho_0\, g\, T_r\left(1+\bar{\lambda}\frac{\partial}{\partial t}\right)\nabla_1^2\theta' - \frac{\mu}{k}\nabla^2 w' \end{aligned} \tag{23}$$

$$\gamma\frac{\partial\theta'}{\partial t} + w'\frac{d\theta_b}{dz} = \kappa\,\nabla^2\theta' + \delta\, exp\left(\theta_b\right)\theta' \tag{24}$$

where $\nabla_1^2 = \left(\partial^2/\partial x^2\right) + \left(\partial^2/\partial y^2\right)$. Using the scaling

$$w^* = \frac{w' d}{\kappa},\ t^* = \frac{t'\kappa}{\varepsilon d^2},\ z^* = \frac{z}{d} \tag{25}$$

(23) and (24) can be expressed (after dropping the asterisks) in the following dimensionless form

$$\begin{aligned} &\left(1+\Gamma\frac{\partial}{\partial t}\right)\left[\frac{1}{Pr_D}\frac{\partial}{\partial t}\left(\nabla^2 w\right)\right] + \nabla^2 w \\ &\quad = Ra\left(1+\Gamma\frac{\partial}{\partial t}\right)\nabla_1^2\theta\ , \end{aligned} \tag{26}$$

$$\frac{1}{\chi}\frac{\partial\theta}{\partial t} + \frac{d\theta_b}{dz}\, w = \nabla^2\theta + F\, exp\left(\theta_b\right)\theta \tag{27}$$

Here $Pr_D = \varepsilon^2\mu d^2/\rho_0 k\kappa$ is the Darcy-Prandtl number, $Ra = \alpha\rho_0 g k T_r\, d/\mu\kappa$ the media Darcy-Rayleigh number, $\Gamma = \bar{\lambda}\kappa/\varepsilon d^2$ the Deborah number and $\chi = \varepsilon/\gamma$ the normalized porosity. ($Pr = \nu/\kappa$ is the Prandtl number and $Da = k/d^2$ is the Darcy number). The buoyancy due to thermal gradient is characterized by the Rayleigh number, the ratio between viscous and thermal diffusivities is given by Prandtl number, the ratio between porosity, Prandtl number and Darcy number is given by Darcy-Prandtl number, the viscoelastic character of the fluid appears in the relaxation parameter Γ, which is also known as the Deborah number. The normalized porosity is expressed as a ratio of porosity of the porous medium and the solid to fluid heat capacity ratio. The range of values for Darcy-Prandtl number for the Maxwell fluid in porous media is not available [15]. Therefore, we consider a wide range of values for the Darcy-Prandtl number.

We now assume that the solutions of (26) and (27) have the form

$$\begin{bmatrix} w' \\ \theta' \end{bmatrix} = \begin{bmatrix} W(z) \\ \Theta(z) \end{bmatrix} exp\left(i(lx+my)+\sigma t\right) \tag{28}$$

where l and m are the horizontal wavenumbers in the x and y directions respectively. The quantity σ refers to the growth rate. Substitution of (28) into (26) and (27) yields the following equations

$$\begin{aligned} &\frac{\sigma}{Pr_D}(1+\sigma\Gamma)\left(D^2-a^2\right)W + \left(D^2-a^2\right)W \\ &\quad + Ra\, a^2\left(1+\sigma\Gamma\right)\Theta = 0 \end{aligned} \tag{29}$$

$$\frac{1}{\chi}\sigma\Theta + \frac{d\theta_b}{dz}W = \left(D^2-a^2\right)\Theta + F\, exp\left(\theta_b\right)\Theta \tag{30}$$

Eqs. (29) and (30) are solved subject to the boundary conditions

$$\left.\begin{aligned} W = \Theta = 0 \quad \text{at} \quad z = 1 \\ W = D\Theta = 0 \quad \text{at} \quad z = 0 \end{aligned}\right\} \tag{31}$$

IV. Method of Solution

The system comprising (29) and (30) and the homogeneous boundary conditions (31) is an eigenvalue problem with Ra being the eigenvalue. An approximate solution of this eigenvalue problem can be obtained by the well-known Galerkin method [24]. To this end, we let

$$W = A_1 W_1 \ \text{ and } \ \Theta = B_1\Theta_1 \tag{32}$$

where W_1 and Θ_1 are the trial functions that must satisfy the boundary conditions (31). Substituting (32) into (29) and (30), multiplying the resulting equations by W_1 and Θ_1 respectively, integrating each equation between $z = 0$ and $z = 1$, and performing some integration by parts, we obtain

$$A_1 X_1\left[\left(1+\sigma\Gamma\right)\left(\frac{\sigma}{Pr_D}\right)X_1+X_2\right] - B_1\left[a^2\,Ra\left(1+\sigma\Gamma\right)Y_1\right]=0 \tag{33}$$

$$X_2 A_1+\left(\frac{\sigma}{\chi}Y_2+Y_3-F\,Y_4\right)B_1=0 \tag{34}$$

where

$$\left.\begin{array}{l} X_1=\left\langle\left(DW_1\right)^2\right\rangle+a^2\left\langle W_1^2\right\rangle \\ X_2=\left\langle W_1\,\dfrac{d\theta_b}{dz}\,\Theta_1\right\rangle \\ Y_1=\left\langle W_1\,\Theta_1\right\rangle,\ Y_2=\left\langle\Theta_1^2\right\rangle \\ Y_3=\left\langle\left(D\Theta_1\right)^2\right\rangle+a^2\left\langle\Theta_1^2\right\rangle \\ Y_4=\left\langle\Theta_1\,e^{\theta_b}\,\Theta_1\right\rangle \\ \langle\, f\,\rangle=\int_0^1 f(z)\,dz \end{array}\right\} \tag{35}$$

The criterion for the existence of the unique solution of the system of (33) and (34) leads to

$$Ra=\frac{X_1\left[\left(1+\sigma\Gamma\right)\dfrac{\sigma}{Pr_D}+1\right]\left(\dfrac{\sigma}{\chi}Y_2+Y_3-F\,Y_4\right)}{a^2\,X_2\left(1+\sigma\Gamma\right)Y_1} \tag{36}$$

The marginal Ra corresponding to the onset of stationary convection (when the principle of exchange of stabilities is valid) is obtained from (36) by taking $\sigma=0$ as

$$Ra_S=\frac{X_1\left(F\,Y_4-Y_3\right)}{a^2\,X_2\,Y_1}\,. \tag{37}$$

Keeping in mind the chosen boundary conditions (31), we deal with the following trial functions

$$\left.\begin{array}{l} W_1(z)=\sin(\pi z) \\ \Theta_1(z)=\cos(\dfrac{\pi}{2}z) \end{array}\right\} \tag{38}$$

The trial functions for the *z*-component of velocity and temperature satisfy the boundary conditions, but may not exactly satisfy the differential equations. This results in residuals when the trial functions are substituted into the differential equations. The Galerkin method warrants the residuals be orthogonal to each trial function.

We notice that the viscoelastic parameter Γ is associated with σ. Thus, when $\sigma=0$, there is no distinction between viscous and viscoelastic fluids. Thus we are interested here only in the onset of oscillatory convection for which case we put $\sigma=i\omega$ in (36), where ω is real. As Ra is real this leads to the expressions for the frequency and marginal Ra corresponding to the oscillatory state as

$$\omega^2=\frac{\chi\left(Y_3-FY_4+F\Gamma Pr_D Y_4-\Gamma Pr_D Y_3\right)+Pr_D Y_2}{\chi\Gamma^2\left(FY_4-Y_3\right)} \tag{39}$$

and

$$Ra_O=\frac{-X_1\left[Pr_D Y_2^2+\chi\left(Y_3-F\,Y_4\right)\left\{Y_2+\chi\left(Y_3-F\,Y_4\right)\right\}\right]}{a^2\chi^2\Gamma^2 Pr_D X_2\,Y_1\left(Y_3-F\,Y_4\right)} \tag{40}$$

V. Results and Discussion

Using the trial functions in (38) we obtain the marginal Ra corresponding to stationary convection as

$$Ra_S=\frac{-3\pi}{64}\,\frac{\left(\pi^2+a^2\right)\left(\pi^2+4a^2-8FY_4\right)}{a^2\,X_2} \tag{41}$$

The minimum value of the Rayleigh number occurs at

$$a_C{}^2=\left[\frac{\pi}{2\sqrt{2}}\sqrt{\pi^2-8FY_4}\right] \tag{42}$$

This is horizontal wave number at the onset of free convection. (42) is obtained by differentiating (41) with respect to *a*. Substituting (42) in (41) gives the value of the critical Rayleigh number. This expression for a_C is same as obtained previously by Malashetty *et al.* [19].

The media Darcy-Rayleigh number for the stationary case, as seen in (37) is a function of the Frank-Kamanetskii number and wave number and it is independent of the viscoelastic parameter, normalized porosity and Darcy-Prandtl number. This is because of the absence of base flow in the present case. In order to illustrate the effect of Frank-Kamanetskii number on the exchange of stabilities and overstability, we evaluate critical media-Darcy Rayleigh number for both stationary and oscillatory cases. Examining Table 2, it is clear that the overstability occurs before that of the exchange of stability *i.e.*, $Ra_O^c<Ra_S^c$ for the same wave number.

Table 2. Comparison of critical values for stationary and oscillatory cases with $\Gamma = 0.5$, $Pr_D = 10$, $\chi = 0.5$.

F	Ra_S^c	a_S^c	Ra_O^c	a_O^c	ω_c^2
0.1	682.978	2.1975	268.547	3.80299	11.2453
0.2	319.07	2.17018	127.109	3.80525	11.2174
0.3	197.162	2.13843	79.7518	3.80791	11.1862
0.4	135.486	2.10049	55.8166	3.8111	11.1506
0.5	97.8329	2.05361	41.231	3.81504	11.109
0.6	71.91	1.99233	31.2232	3.82012	11.0586
0.7	52.2446	1.90405	23.6823	3.82721	10.9935
0.8	35.3437	1.74458	17.3102	3.83885	10.8964
0.8785	13.4634	0.545699	10.374	3.87177	10.6876

It is also seen from Table 2 that both Ra_O^c and Ra_S^c decrease as the value of F is increased. Thus the effect of chemical reaction in a horizontal porous layer containing a viscoelastic fluid is to enhance instability.

The critical wavenumber of the oscillatory convection is larger than that of stationary convection, just as in the case of a Newtonian fluid saturated porous layer [15].

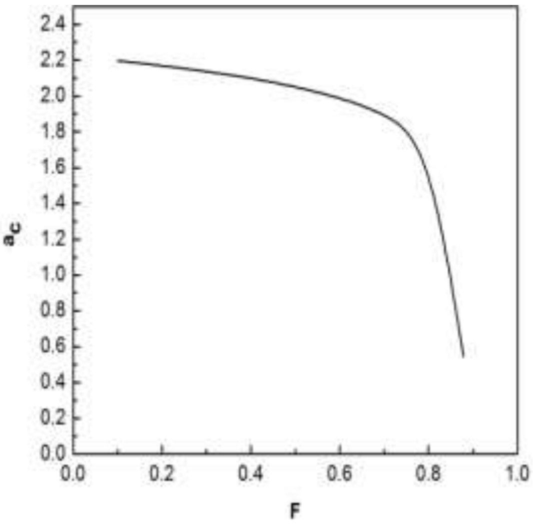

Figure 4. Variation of critical wave number with Frank-Kamenetskii number for stationary case.

Further, as seen in Fig. 4, the decrease in the critical wave number for the stationary case is drastic near the ignition point. It is important to note that stationary convection is independent of the viscoelastic parameter and the result is same as in case of a Newtonian fluid undergoing a zero order chemical reaction.

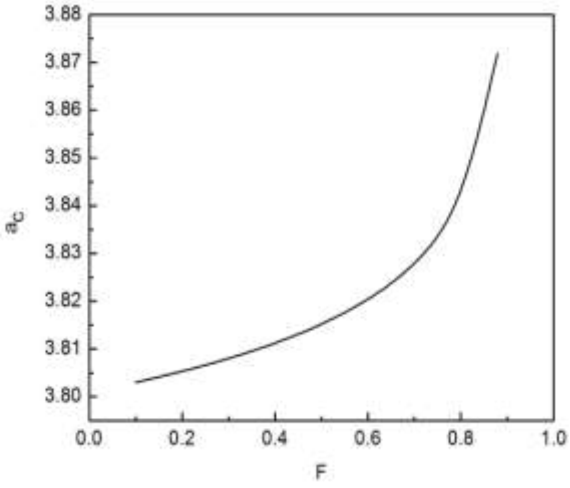

Figure 5. Variation of critical wave number with Frank-Kamenetskii number for oscillatory case.

For the oscillatory case, the critical wave number increases with increasing F and the increase is drastic near the ignition point as seen in Fig. 5. It is also seen that the angular frequency of the system reduces significantly with F.

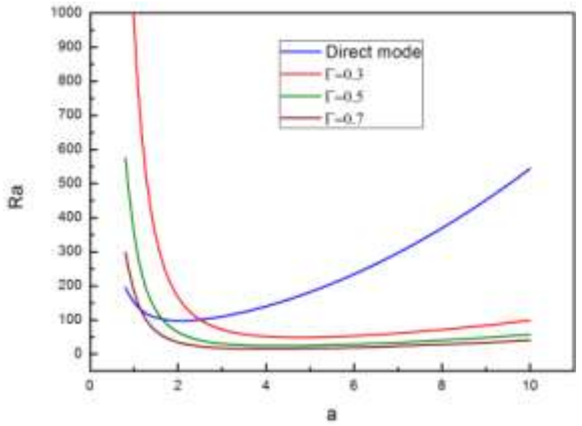

Figure 6. Stability curves showing *Ra*, as a function of wave number and its dependence on Deborah number Γ for $F=0.5,\ \chi=0.5,\ Pr_D=20$.

In order to illustrate the effect of relaxation time of the fluid on the overstability, we plot the typical curves of the Rayleigh number as a function of the wave number in Fig. 6. Examining, we observe that the Maxwell fluid with a higher value of the relaxation time will exhibit overstability at a lower Rayleigh number. As the Deborah number increases, the critical Rayleigh number for overstability decreases. This indicates that elasticity of a Maxwell fluid has a destabilizing influence on a fluid layer undergoing a chemical reaction in a porous medium heated from below.

The effect of Prandtl number is also important because many practical viscoelastic liquids have large Prandtl number. In Fig. 7 the effect of Darcy-Prandtl number on the stability of the system is displayed. For the same wave number, the overstability curves lie far below that of exchange of stability.

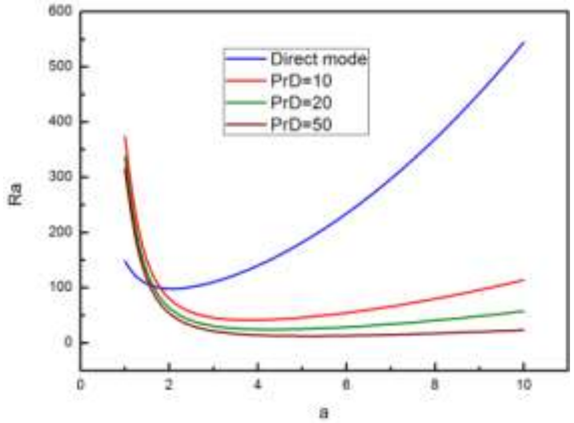

Figure 7. Stability curves showing *Ra* as a function of wave number and it dependence on Darcy-Prandtl number Pr_D for $F=0.5, \chi=0.5, \Gamma=0.5$.

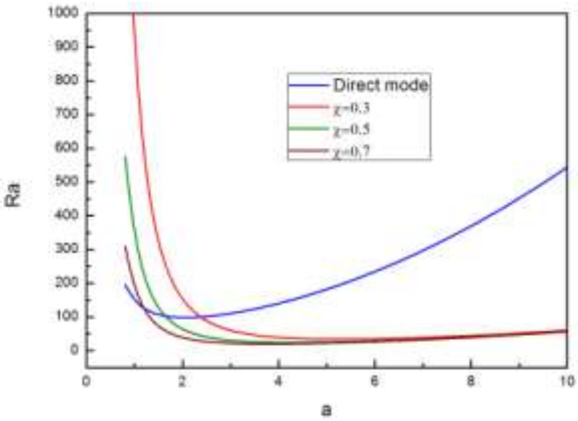

Figure 8. Stability curves showing *Ra* as a function of wave number and it dependence on normalized porosity χ for $F=0.5,\ Pr_D=20, \Gamma=0.5$.

Thus Darcy-Prandtl number, which is the ratio involving porosity, Prandtl number and Darcy number, has a destabilizing influence on the system. Further, all the neutral stability curves of oscillatory mode for different Pr_D remain apart in the large wavenumber regions. This behaviour has been observed before for a Maxwell fluid in non-porous cases [13, 25].

Fig. 8 shows the effect of normalised porosity on the neutral curves. It is observed that with the increasing value of χ the values of *Ra* for the oscillatory mode decreases. This indicates that the effect of increasing normalized porosity parameter is to destabilize the system.

We also observe that the overstability curves have a very flat bottom, indicating instability can occur within a broad wave number band. This behaviour was also observed by Khudeja *et al.* [21] for a chemically reactive Maxwell fluid having an isothermal lower boundary. Further we observe that all the neutral curves for overstability for different χ coalesce in the large wave number region.

Furthermore, it is noted that the critical wave number, representing the size of cell pattern at the onset of stability, for the direct mode is smaller than that of oscillatory mode. The critical wave number decreases with increase in F, χ and Γ and increases with increase in Pr_D. The critical frequency increases with increase in Pr_D and F and it decreases with increase in χ and Γ.

VI. Conclusions

In this study, based on the modified-Darcy-Maxwell model, linear convective instability of a Maxwell fluid layer undergoing a zero-order exothermic chemical reaction in a densely packed porous medium with constant heat flux lower boundary has been analysed. The effect of chemical reaction has been dealt with in great detail and the critical value of the parameter corresponding to chemical reaction has been determined. The critical media Darcy-Rayleigh number, wave number and frequency for overstability are determined.

It is observed that there is competition amongst the processes of viscous relaxation, chemical reaction and thermal diffusion that causes the first convective instability to be oscillatory than stationary convection. The result of the study carried out can be summarized as follows:

(i) The Maxwell fluid behaves like an ordinary Newtonian fluid undergoing a chemical reaction for the case where exchange of stabilities is valid.
(ii) The stationary mode is independent of the viscoelastic parameter. The viscoelastic properties of the fluid are observed only in the case of oscillatory convection.
(iii) The zero order chemical reaction taking place is seen to destabilize the system.
(iv) The effect of increasing stress relaxation time is to destabilise the system.
(v) The Darcy-Prandtl number and normalized porosity hasten the onset of convection.
(vi) The dimension of the convection cells is influenced by the presence of chemical reaction, stress relaxation time, porosity of the medium and the Darcy-Prandtl number.

References

[1] C.M. Vest and V.S. Arpaci, Overstability of a viscoelastic fluid layer heated from below, *J. Fluid Mech., 36*, 1969, 613-623.
[2] M. Sokolov and R.I. Tanner, Convective stability of a general viscoelastic fluid heated from below, *Phys. Fluids, 52*, 1972, 321-327.
[3] S. Rosenbalt, Thermal convection in a viscoelastic liquid, *J. Non-Newt. Fluid Mech., 21*(*2*), 1986, 201-223.
[4] J. Martínez-Mardones and C. Pérez-García, Bifurcation analysis and amplitude equations for viscoelastic convective fluids, *Il Nuovo Cimento D, 14*(*9*), 1992, 961-975.
[5] D.A. Nield and A. Bejan, *Convection in porous Media* (New York: Springer, 2006).
[6] M. Kaviany, *Principles of heat transfer in porous media* (New York: Springer, 1991).
[7] D.B. Ingham and I. Pop, *Transport phenomena in porous media* (Oxford: Pergamon, 1998).
[8] K. Vafai, *Handbook of porous media* (New York: Marcel Dekker, 2000).
[9] D.M. Herbert, On the stability of viscoelastic liquids in heated plane Couette flow, *J. Fluid Mech*., *17*, 1963, 353-359.
[10] T. Green, Oscillating convection in an elasticoviscous liquid, *Phys. Fluids, 11*, 1968, 1410-1412.
[11] M.C. Kim, S.B. Lee, S. Kim and B.J. Chung, Thermal instability of viscoelastic fluids in porous media, *Int. J. Heat Mass Transf., 46*, 2003, 5065-5072.
[12] M.S. Malashetty, M.S. Swamy and W. Sidram, Thermal convection in a rotating viscoelastic fluid saturated porous layer, *Int. J. Heat Mass Transfer, 53*, 2010, 5747-5756.
[13] W.C. Tan and T. Masuoka, Stability analysis of a Maxwell fluid in a porous medium heated from below, *Phys. Lett. A, 360*, 2007, 454-460.
[14] S.W. Wang and W.C. Tan, Stability analysis of double-diffusive convection of Maxwell fluid in a porous medium heated from below. *Phys. Lett. A, 372*, 2008, 3046-3050.
[15] M.S. Malashetty and I. Begum, Effect of thermal/gravity modulation on the onset of convection in a Maxwell fluid saturated porous layer, *Transp. Porous Med., 90*, 2011, 889-909.
[16] H.V. Ersoy, Flow of a Maxwell fluis in a porous orthogonal rheometer under the effect of a magnetic field, *Amer. J. Mech. Engg., 3*(*1*), 2016, 16-20.
[17] W. Kordylewski and Z. Krajewski, Convection effects on thermal ignition in porous media. *Chem. Engg. Sci., 39,* 1984, 610-612.
[18] W.W. Farr, J.F. Gabitto, D. Luss and V. Balakotaiah, Reaction-driven convection in a porous medium. *A.I.Ch.E.J., 37,* 1991, 963-985.
[19] M.S. Malashetty, P. Cheng and B.H. Chao, Convective instability in a horizontal porous layer saturated with a chemically reacting fluid. *Int. J. Heat Mass Transfer, 37,* 1994, 2901-2908.
[20] G. McKay, Onset of buoyancy driven convection in superposed reacting fluid and porous layers, *J. Engg. Math., 33,* 1998, 31-46.
[21] S. Khudeja Akbar, A.L. Nargund and S. Maruthamanikandan, Convective

instability in a horizontal porous layer saturated with a chemically reacting Maxwell fluid, *AIP Conf. Proc.*, *1557*, 2013, 130-136.

[22] M.M. Rashidi, N. Freidoonimehr, A. Hosseini, O. Anwar Beg and T.K. Hung, Homotopy simulation of nanofluid dynamics from a non-linearly stretching isothermal permeable sheet with transpiration, *Meccanica*, *49*(*2*), 2014, 469-482.

[23] Mubeen Taj, S. Maruthamanikandan, S. Khudeja Akbar, Effect of chemical reaction on convective instability in a horizontal porous layer saturated with a couple-stress fluid, *Int. J. Engg. Res. Appl.*, *3*(*2*), 2013, 1742-1748.

[24] B.A. Finlayson, *The method of weighted residuals and variational principles* (New York: Academic Press, 1972).

[25] S. Maruthamanikandan, *Instabilities in ferromagnetic, dielectric and other complex liquids*, Ph.D. Thesis, Bangalore University, India, 2005.